1990 – 1983

LEAVING CERTIFICATE EXAMINATION, 1990

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 22 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle is projected vertically upwards with velocity u m/s and is at a height h after t_1 and t_2 seconds respectively. Prove that

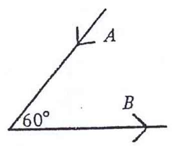
$$t_1 \cdot t_2 = \frac{2h}{g}$$

(b) A car accelerates uniformly from rest to a speed v m/s. It continues at this constant speed for t seconds and then decelerates uniformly to rest.
The average speed for the journey is $\frac{3v}{4}$.

(i) Draw a speed-time graph and hence, or otherwise, prove that the time for the journey is $2t$ seconds.

(ii) If the car-driver had observed the speed limit of $\frac{1}{2}v$, find the least time the journey would have taken, assuming the same acceleration and deceleration as in (i).

2. Two straight roads intersect at an angle of 60°.
Car A moves towards the junction with uniform speed 16 m/s, while car B moves away from the junction with uniform speed 20 m/s.



(a) Calculate the velocity of A relative to B.

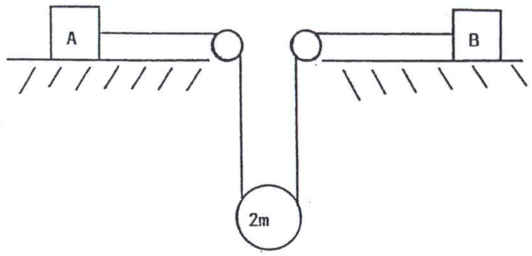
(b) If A is 450 m and B is 200 m from the intersection at a given moment, calculate the time interval in seconds until the cars
(i) are nearest to each other
(ii) are equidistant from the intersection.

3. A particle is projected from a point p, up a plane inclined at an angle $\tan^{-1}\frac{1}{6}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope.)

(i) If the particle were to strike the inclined plane horizontally at a point q, show that $\tan\alpha = \frac{3}{19}$.

(ii) If the particle were to be projected from p with the same speed but at an angle $\tan^{-1} 3$ to the inclined plane, show that it would strike the plane at right angles at q.

4. Two blocks A and B each of mass m kg, lie at rest on horizontal rough tables. The coefficient of friction between A and the table is μ, and between B and its table is $\frac{1}{4}$. The blocks are connected by a light inextensible string which passes under a smooth movable pulley of mass $2m$ kg.



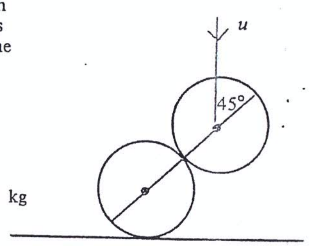
(i) Show in a diagram the forces acting on each mass when the system is released from rest.

(ii) If $\mu < \frac{3}{4}$, prove that the tension in the string is $\frac{mg(9 + 4\mu)}{16}$

(iii) Prove that A will not move if $\mu > \frac{3}{4}$.

5. State the laws governing the oblique collision of two smooth elastic spheres.

A smooth elastic sphere of mass 6 kg rests on a smooth horizontal table. A second smooth elastic sphere of mass 4 kg falls vertically on it. At the moment of impact the line of centres makes an angle of 45° with the vertical, and the velocity of the falling sphere is u. The 6 kg sphere moves horizontally after the collision.



(i) Explain why the principle of conservation of momentum may be applied horizontally.

(ii) Hence, or otherwise, prove that the speed of the 6 kg mass after impact is $\frac{u(1 + e)}{4}$ where e is the coefficient of restitution between the two spheres.

(iii) If $e = \frac{1}{3}$, prove that the loss of kinetic energy due to the impact is $\frac{2u^2}{3}$

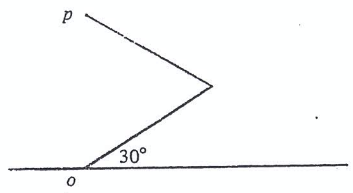
6. (a) A particle starts from rest, and moves with simple harmonic motion of period $6n$ seconds. Show that the particle moves from the position of maximum velocity to the position in which the velocity is half the maximum in n seconds.

(b) The depth of water in a harbour is assumed to rise and fall with time in simple harmonic motion. On a certain day the low tide had a height of 13 m at 12.58 p.m. and the following high tide had a height of 18 m at 6.58 p.m.

If a ship requires a depth of $16{\cdot}5$ m of water before it can leave the harbour, find the latest time on that day that the ship can leave the harbour.

7. (a) Define
(i) limiting friction,
(ii) coefficient of friction,
(iii) angle of friction.

(b) A uniform rod of length $2a$ rests on a rough horizontal plane at point o, and is held inclined at an angle of 30° to the horizontal by a string tied at its top end and to a fixed point p distant $2a$ vertically above o.



If the rod is on the point of slipping, calculate the coefficient of friction between the rod and the plane.

8. (a) Prove that the moment of inertia of a uniform square lamina, of mass m and side $2a$, about an axis through its centre parallel to one of its sides is $\frac{1}{3}ma^2$.

(b) A square lamina p, q, r, s can turn freely about a horizontal axis through p perpendicular to the plane of the lamina.

(i) If the lamina is released from rest when diagonal pr is horizontal, find its angular velocity when pr is vertical.

(ii) What mass must be attached to the lamina at r so that the combined body will oscillate with period (of small oscillations)

$$2\pi\sqrt{\frac{8a}{3g}}$$

9. (a) State Archimedes' Principle.
A plastic block of volume 330 cm³ has air bubbles in it. The block floats in water with 80% of its volume immersed. If the relative density of the plastic is 1·2, calculate the volume of the air bubbles.

(b) A cubical block of wood of side 10 cm and of relative density 0·6 floats horizontally in a container of water. Oil of relative density 0·8 is poured on the water until the top of the oil layer is 3 cm below the top of the block.

(i) How deep is the layer of oil ?
(ii) What is the pressure on the lower face of the block ?

10. (a) Solve the differential equation

$$x\frac{dy}{dx} = y(1 + y)$$

if $x = 1$ when $y = 1$.

(b) A particle of mass 8 kg starts from rest and is acted on by a force which increases uniformly in 10s from zero to 16N.

(i) Prove that t seconds after the particle begins to move, its acceleration is $\frac{t}{5}$ m/s².

(ii) Prove that, when the particle has moved x m, its speed is v m/s, where $10v^3 = 9x^2$.

LEAVING CERTIFICATE EXAMINATION, 1989

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 23 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s².
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. Two cars A and B, each 5 m in length, travel with constant velocity 20 m/s along a straight level road. The front of car A is 15 m directly behind the rear of car B. Immediately on reaching a point P each car decelerates at 4 m/s².

(i) Show that A collides with B.
(ii) At what distance from P does the collision occur ? 25 7/8 m.
(iii) Show the motion of both cars on the same speed-time graph.

2. A man travelling due North at 20 m/s finds that the wind appears to blow from the West. When he travels due West at 8·45 m/s the wind appears to blow from the South West.

(i) Calculate the velocity of the wind.
(ii) If the man travelled in a direction 30° North of West at 8 m/s from what direction would the wind appear to blow ?

3. A particle is projected with speed u at an angle α to the horizontal. The range of the particle on the horizontal plane through the point of projection is R.

(i) Show that R is a maximum when $\alpha = 45°$.
(ii) If $R = \frac{u^2}{2g}$, find the two possible values of α.
(iii) If the ratio of the greatest height to the range is 2 : 5, find α.

4. State the laws governing the oblique collision of elastic spheres.
A smooth sphere A, of mass m, moving with speed 0·6 m/s, impinges obliquely on a smooth sphere B, of mass 2 m, which is at rest. After the collision A is found to move with speed 0·2 m/s in a direction at right angles to its original direction.

(i) Find the direction of A before impact.
(ii) Find the coefficient of restitution.
(iii) Show that the loss of kinetic energy, as a result of the impact, is 0·06 m.

5. A wedge of mass 8 kg can slide freely on a smooth horizontal table. On one face, inclined at an angle of 30° to the horizontal, is placed a particle of mass 4 kg, and on the other face, inclined at an angle of 60° to the horizontal, is placed a particle of mass 6 kg. If both faces of the wedge are smooth,

(i) Show on separate diagrams the forces acting on each mass.
(ii) Show that when the particles are released from rest, the acceleration of the wedge is $\frac{g}{9\sqrt{3}}$

6. Define Simple Harmonic Motion.

A mass of 4 kg suspended by a light spiral spring extends it 8 cm when in equilibrium. A second mass of 2 kg is attached to the first without moving it and the combined mass is then released from rest.

(i) Prove that the motion is simple harmonic.

(ii) Find the periodic time of the ensuing motion.

(iii) Find the maximum velocity of the resulting motion.

7. A particle of mass 10 kg is placed on a rough inclined plane. The least force acting up along the plane which will prevent the particle slipping down the plane is 19·6 N. The least force acting up along the plane which will make the particle slip upwards is 98 N.

(i) Find the inclination of the plane.

(ii) Show that the coefficient of friction is $\frac{1}{2}$.

(iii) Find the least force required to move the particle up the plane. The least force need not necessarily be parallel to the plane.

8. Prove that the moment of inertia of a uniform circular lamina of mass M and radius r about an axis through its centre, perpendicular to the plane of the lamina, is $\frac{1}{2}Mr^2$.

A circular sheet of cardboard of radius r rotates freely in its own plane, which is vertical, about a horizontal pin. At what distance from the centre should the pin be stuck to make the period of small oscillation a minimum ?

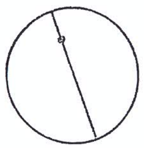


fig. 1

9. (a) A cube of side 1 m is filled to a height of x m with water and a second liquid of relative density 0·8, which does not mix with water, occupies the remainder of the cube. The thrust on each vertical side due to the water is equal to the thrust due to the other liquid. Find x.

(b) Two solid uniform spheres each of radius 6 cm are connected by a light string and are completely immersed in a tank of water. The heavier sphere lies on the bottom of the tank. The relative densities of the spheres are 0·75 and 2·25 respectively.
Find the tension in the string and the reaction between the bottom of the tank and the heavier sphere.

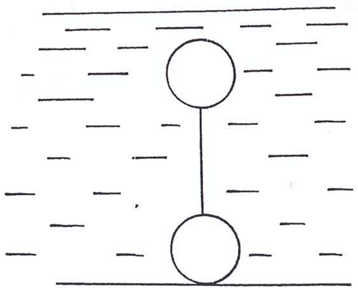


fig. 2

10. (a) Find the solution of the differential equation

$$x\frac{dy}{dx} = y + xy$$

if $y = 1$ when $x = 1$.

(b) A cyclist, free-wheeling on a straight level road, experiences a retardation which is proportional to the square of his speed. His speed is reduced from 6 m/s to 3 m/s in a distance of 35 m.

Show that the average speed during this period is $6 \; ln \; 2$.

LEAVING CERTIFICATE EXAMINATION, 1988

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 17 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle moving in a straight line with uniform acceleration describes 23 m in the fifth second of its motion and 31 m in the seventh second. Calculate its initial velocity. a = 4 m s^{-2} u = 5 m s^{-1}

(b) A particle falls freely from rest from a point o, passing three points a, b and c, the distances ab and bc being equal. If the particle takes 3 s to pass from a to b and 2 s from b to c, calculate $|ab|$. 15g m

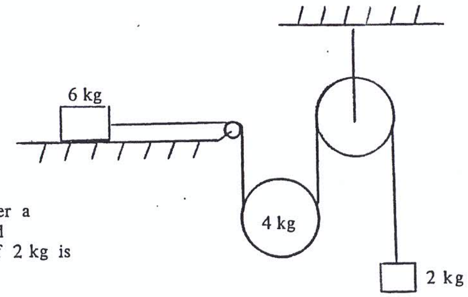
2. (a) Two boats move with constant speed 5 m/s relative to the water and both cross a straight river of width 72 m flowing with constant speed 3 m/s parallel to the banks. One crosses by the shortest path and the other in the shortest time. Show that the difference in the times taken is 3·6 s.

(b) Two ships A and B move with constant speeds $2u$ and u respectively. At a certain instant, B is 2400 m due east of A and moving northwards. Show that A must move in the direction 30° North of East in order to intercept B and find (in terms of u) the time it takes to intercept B.

3. (a) A particle which is projected with speed u has a horizontal range $\frac{3u^2}{49}$.
Calculate the two possible angles of projection.

(b) A particle is projected up an inclined plane with initial speed $13u$. The line of projection makes an angle $\tan^{-1}(\frac{5}{12})$ with the plane and the plane is inclined at 45° to the horizontal. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane at a point p.
If the coefficient of restitution between the particle and the plane is 0·4, show that the particle rises vertically from p and strikes p again on the second bounce.

4. One end of a light inextensible string is attached to a mass of 6 kg which rests on a rough horizontal table.
The coefficient of friction between the mass and the table is $\frac{1}{6}$.
The string passes over a smooth fixed pulley at the edge. Then it passes under a smooth movable pulley of mass 4 kg and over a smooth fixed pulley. A mass of 2 kg is attached to its other end.

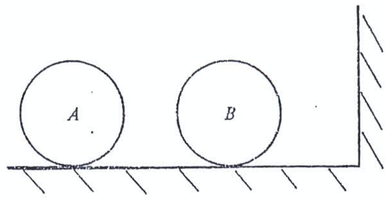


(i) Show on separate diagrams the forces acting on each mass.

(ii) Calculate the acceleration of each mass and the tension in the string in terms of g, the acceleration due to gravity.

1988

5. Two smooth spheres, A and B, of equal radii, have masses 4 kg and 8 kg respectively. They lie at rest on a smooth horizontal floor so that the line joining their centres is perpendicular to the vertical wall.



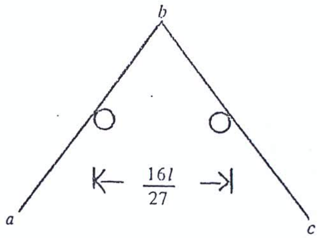
A is projected towards B with speed u and collides with B. B then hits the wall, rebounds and collides with A again. This final collision reduces B to rest. If the coefficient of restitution between A and B is $\frac{1}{4}$, calculate

(i) the coefficient of restitution between B and the wall.
(ii) the final velocity of A in terms of u.
(ii) the total loss of energy due to the three collisions.

6. A particle of mass 8 kg is describing a circle, with constant speed v, on a smooth horizontal table. It is connected by a light inextensible string of length 3 m to a point which is 1 m vertically above the centre of the circle.

(i) Calculate the tension in the string.
(ii) Show that the particle will remain in contact with the table if $v \leqslant \sqrt{8g}$.
(iii) If the speed of the particle is increased to $\sqrt{9{\cdot}1g}$, calculate the height at which the particle rotates above the table.

7. Two equal uniform rods ab and bc each of length $2l$ and weight W, are freely jointed at b and rest in equilibrium, in a vertical plane, across two smooth horizontal pegs at the same horizontal level and distant $\frac{16l}{27}$ apart.



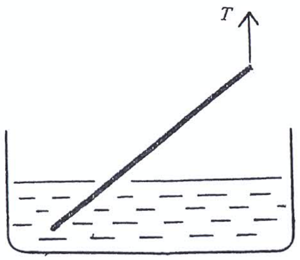
(i) Show in separate diagrams the forces acting on each rod.
(ii) Show that the inclination of each rod to the vertical is $\sin^{-1}(\frac{2}{3})$.
(iii) Determine the magnitude and direction of the reaction at b.

8. Show that the moment of inertia of a uniform rod of mass m and length $2l$, about an axis through its centre of mass perpendicular to the rod is $\frac{1}{3}ml^2$.

Three of these rods are joined together at their ends to form a triangle abc. The triangle is free to rotate about a fixed horizontal axis through a, perpendicular to its plane. Find the period of small oscillations about the equilibrium position.

9. State the Principle of Archimedes.

A uniform rod of weight W and of length $2l$, in equilibrium, is supported at one end by a vertical force T and is immersed in water as shown in the diagram.
The relative density of the rod is $\frac{7}{16}$.



(i) Calculate the length of the immersed part of the rod.
(ii) Show that $T = \frac{3W}{7}$.

10. (a) Solve the differential equation

$$\frac{dx}{dt} = \sqrt{100 - 4x^2}$$

if $x = 5$ when $t = 0$.

(b) A particle of mass m is projected vertically upwards with speed 120 m/s in a medium where there is a resistance of $0{\cdot}098\,v^2$ per unit mass of the particle when v is the speed. Calculate the time taken to reach the highest point.

LEAVING CERTIFICATE EXAMINATION, 1987

APPLIED MATHEMATICS – HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) The maximum acceleration of a body is 4 m/s^2 and its maximum retardation is 8 m/s^2. What is the shortest time in which the body can travel a distance of 1200 m from rest to rest ?

30 secs.

(b) A car A starts from a point p with initial velocity of 8 m/s and then travels with a uniform acceleration of 4 m/s^2. Two seconds later a second car B starts from p with an initial velocity of 30 m/s and then moves with a uniform acceleration of 3 m/s^2.

Show that after passing A, B will never be ahead by more than 74 m.

2. At a certain instant a ship H is 37·5 km due West of a ship K. Ship H is travelling South-East at 25 km/h and ship K is travelling South at 15 km/h.

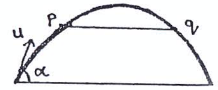
(i) Draw a diagram to show the velocity of K relative to H and calculate the magnitude and direction of the relative velocity.

(ii) If H and K can exchange signals when they are not more than 20 km apart, calculate when they can begin to exchange signals and for how long they can continue to exchange signals.

3. (a) A particle is projected up a plane, which is inclined at an angle $\tan^{-1}\frac{1}{4}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). If the particle strikes the inclined plane at right angles, show that $\tan\alpha = 2$.

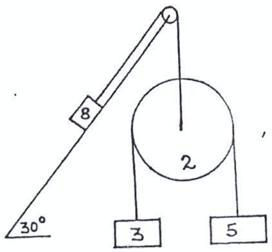
1987

(b) A particle is projected with speed u at an angle α to the horizontal. The particle takes 4 s to travel between two points p and q which are on the same horizontal line.



Show that the greatest height the particle reaches above this line is 19·6 m.

4. A particle of mass 8 kg rests on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is $\frac{1}{\sqrt{3}}$. The 8 kg mass is connected, by a light inextensible string passing over a smooth fixed pulley at the top of the plane, to a pulley of mass 2 kg hanging freely. Over this pulley which is also smooth, a second light inextensible string is passed having particles of mass 3 kg and 5 kg respectively, attached.



(i) Show in a diagram the forces acting on each mass when the system is released from rest.

(ii) Calculate the acceleration of the 8 kg mass.

5. State the laws governing the oblique collision of smooth elastic spheres.

Two smooth elastic spheres A and B of mass 4 kg and 8 kg respectively, collide obliquely. The coefficient of restitution is 0·4. Before collision the velocity of A is $(3\vec{i} + 4\vec{j})$ m/s and that of B is $(-4\frac{1}{2}\vec{i} - p\vec{j})$ m/s where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the line of centres at the moment of impact.

(i) Find the velocity of each sphere after impact.
(ii) Show that the loss of kinetic energy, as a result of the impact, is 63 J.
(iii) If after impact the spheres are moving at right angles to each other, calculate the value of p.

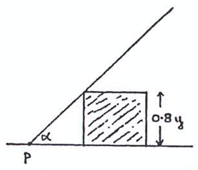
6. Define simple harmonic motion.

A particle of mass m is suspended from a fixed point p by a light extensible string of natural length d and elastic constant $\frac{49m}{d}$. It is pulled down a distance $\frac{8d}{5}$ below p and is then released from rest.

(i) Show that the particle moves with simple harmonic motion as long as the string remains taut.
(ii) Find in terms of d, when the string becomes slack for the first time.

7. Define limiting friction and coefficient of friction.

A uniform rod of mass 2 kg and of length $6y$ metres, leans against the smooth edge of a rectangular block of mass 6 kg and height $0{\cdot}8y$ metres. The rod is smoothly hinged at p to a rough horizontal floor and the block also rests on the floor (see diagram). The block is on the point of slipping when the rod makes an angle α with the horizontal, where $\tan \alpha = \frac{4}{3}$.

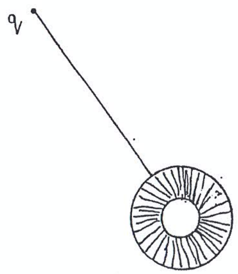


(i) Show in separate diagrams the forces acting on the rod and on the block.
(ii) Show that the coefficient of friction between the block and floor is $\frac{6}{17}$.
(iii) Find, correct to the nearest Newton, the magnitude of the reaction at the hinge.

8 Prove that the moment of inertia of a uniform annulus of internal diameter p, external diameter $3p$ and mass $4m$, about an axis through its centre perpendicular to its plane is $5mp^2$. (see Tables P.40)

A uniform rod of mass m and length $6p$ is attached to the rim of this annulus so that the rod and the annulus are in the same plane and the rod is collinear with a diameter of the annulus (see diagram).

If the compound body is set in motion about an axis through q which is perpendicular to the plane of the rod and the annulus,



(i) find the period of small oscillations
(ii) show that the length of the equivalent simple pendulum is $\frac{22p}{3}$.

9. (a) A wooden cube of side 10 cm, and relative density 0·8, is floating horizontally in water. What mass of aluminium, whose relative density is 2·8, must be attached to

(i) the upper face, so that the cube will just be completely immersed horizontally with the aluminium above water?
(ii) the lower face, so that the cube is just immersed and horizontal ?

(b) A uniform rod in equilibrium is inclined to the horizontal with one fifth of its length immersed in a liquid and its upper end supported by a vertical force P.

(i) Show in a diagram the forces acting on the rod.
(ii) If the relative density of the rod is 0·72, calculate the relative density of the liquid.

10. (a) Solve the differential equation

$$2x(1 + y)\frac{dx}{dy} = 8 + x^2$$

if $x = 2$ when $y = 3$.

(b) The resistance to the motion of a train of mass m is constant and equal to 60 N per tonne. When moving with constant speed 16 m/s on a level line the train begins to ascend an incline of 1 in 98, i.e. $\sin^{-1}(1/98)$. Assuming that the engine continues to work at the same rate (i.e. power is constant) and the v m/s is the speed of the train up the incline t seconds after the train has begun climb, show the equation of motion is

$$\left(\frac{v}{v - 6}\right)\frac{dv}{dt} + \frac{4}{25} = 0$$

Calculate the time which elapses before the velocity falls to 12 m/s.
[Tables P.29 may be needed.]

AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1986

APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 27 JUNE - AFTERNOON, 2.00 - 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9 \cdot 8$ m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle with speed 150 m/s begins to decelerate uniformly at a certain instant while another particle starts from rest 8 s later and accelerates uniformly. When the second particle has travelled 135 m both particles have a speed of 30 m/s.

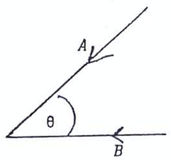
   (i) Show the motion of both particles on the same speed-time graph.

   (ii) How many seconds after the commencement of deceleration does the first particle come to rest ?

   (b) A particle starting from rest at p moves in a straight line to q with uniform acceleration. In the first second it travels 5 m. In the last three seconds of its motion before reaching q it travels $\frac{9}{25}$ of $|pq|$.

   Find the time in seconds from p to q.

2. Two straight roads intersect at an angle θ, $\tan\theta = \frac{4}{3}$. Cars A and B move towards the point of intersection at 16 m/s and v m/s, respectively. If the magnitude of the velocity of A relative to B is 16 m/s, find v.



   If at a given instant A is 96 m and B is $38 \cdot 4$ m from the intersection, calculate

   (i) the shortest distance between them in their subsequent motion.

   (ii) the distance, to the nearest metre, between the two cars 2 s before the instant when they are nearest to each other.

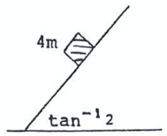
3. A particle is projected with speed u at an angle α to the horizontal. If the maximum height reached is the same as the total horizontal range, show that $\tan\alpha = 4$.

   The particle moves at right angles to its original direction of motion after a time t_1 and then strikes the horizontal plane after 8 s, both times measured from the instant of projection.

   Show $u = g\sqrt{17}$.

   Calculate t_1.

4. A smooth particle of mass 4 m rests on the smooth inclined face of a wedge of mass m and slope $\tan^{-1}2$.



   The wedge is free to move on a rough horizontal table, the coefficient of friction being $\frac{1}{3}$.

   When the system is released from rest, the wedge moves with acceleration p parallel to the table.

   (i) Show on separate diagrams the forces acting on the wedge and on the particle.

   (ii) Calculate p in terms of g.

5. (a) A smooth sphere P of mass $3m$ and velocity $4u$ impinges directly on a smooth sphere Q of mass $5m$ and velocity $2u$, moving in the same direction. The coefficient of restitution is e.

   (i) For what value of e will the velocity of P be halved by the impact ?

   (ii) Show that whatever the value of e in $0 < e < 1$, the velocity of Q after impact exceeds $2u$.

   (b) Two smooth spheres of masses 4 kg and 2 kg impinge obliquely The 2 kg mass is brought to a rest by the impact.

   (i) Prove that, before impact, they were moving in directions perpendicular to each other.

   (ii) Show that, as a result of impact, the kinetic energy gained by the 4 kg mass is equal to half that lost by the 2 kg mass.

6. (a) A particle moves with simple harmonic motion through two points p and q $1 \cdot 2$ m apart. Its velocity is the same at p as at q. It takes 3 s to move from p to q and 3 s to move from q to q i.e. passing q the next time.

   Using a diagram, or otherwise, find the period of the particle and the amplitude.

   (b) An elastic string of natural length $2a$ is stretched between two points which are $2\sqrt{3}a$ apart. A particle of weight W is attached to its midpoint and hangs in equilibrium with the string inclined at an angle of 30° to the horizontal.

   Show that the modulus of elasticity of the string is W.

7. State and prove the relationship between the coefficient of friction, μ, and the angle of friction.

   One end of a uniform rod rests on a rough horizontal floor and the other end rests in contact with a rough vertical wall, the coefficient of friction in both cases being μ. The rod makes an acute angle, θ, with the wall.

   (i) Show in a diagram the forces acting on the rod.

   (ii) Show that when the rod is on the point of slipping

$$\tan\theta = \frac{2\mu}{1 - \mu^2}$$

8. (a) Prove that the moment of inertia of a uniform circular disc, of mass m, and radius r, about an axis through its centre perpendicular to its plane is

$$\tfrac{1}{2}mr^2.$$

   (b) A uniform circular disc, starts from rest and rolls down a plane of inclination 30°. The plane, being rough, prevents sliding.

   Using the principle of conservation of energy, or otherwise, show that the uniform acceleration of the disc is $g/3$ m/s^2.

9. (a) A square plate is immersed vertically in water with an edge of length d in the surface of the water.
   Find how far below the surface is the horizontal line which divides the square into two rectangles on each of which the thrust is the same.

   (b) A hollow spherical shell of external diameter 2 m and uniform thickness $0 \cdot 5$ m, floats in a liquid with half of its volume immersed.
   If the relative density of the liquid is $1 \cdot 4$, find the relative density of the material of the shell.

10. (a) Solve the differential equation

$$x\frac{dy}{dx} = y(1 - x)$$

   if $y = 3$ when $x = 1$.

   (b) A particle moves in a straight line in a medium whose resistance is proportional to the cube of its speed. No other force acts on the body. The speed falls from 15 m/s to $7 \cdot 5$ m/s in a time of t seconds.

   Show that the distance traversed in this time is $10t$ m.

LEAVING CERTIFICATE EXAMINATION, 1985

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 28 JUNE – AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. A bus 12·5 m long travels with constant acceleration. The front of the bus passes a point, p, with speed u while the rear of the bus passes p with speed v.

Find, in terms of u and v

(i) the time taken by the bus to pass p.

(ii) what fraction of the length of the bus passes the point p in half this time.

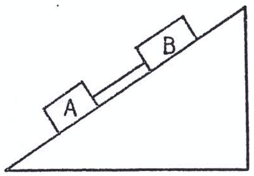
2. A particle is projected from a point o with initial velocity u, up a plane inclined at an angle of 60° to the horizontal. The direction of projection makes an angle θ with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). The maximum height reached above the inclined plane is H.

Express (i) the velocity and displacement from o of the particle after t seconds, in terms of $\vec{i}$ and $\vec{j}$, where these are the unit vectors along and perpendicular to the plane, respectively.

(ii) H, in terms of u and θ.

(iii) the time interval, in terms of $\sin 2\theta$, between the instants when the particle is at a height, $H \sin^2 \theta$, above the inclined plane.

3. Two blocks A and B have masses 2 kg and x kg, respectively. They are connected by a string and slide down an inclined plane which makes an angle $\sin^{-1}(\frac{3}{5})$ with the horizontal. The coefficient of friction between A and the plane is $\frac{1}{4}$, and between B and the plane is $\frac{1}{2}$.



(i) Show in a diagram the forces acting on each block when the system is released from rest.

(ii) Find the acceleration f of the system in terms of x.

(iii) For what value of x would the acceleration of the blocks be $0{\cdot}9f$?

4. State the laws governing the oblique collision of elastic spheres.
A smooth sphere A impinges obliquely on an identical smooth sphere B which is at rest. The direction of A before and after impact makes angles 60° and θ, respectively, with the line of centres.

(i) Prove that $\tan\theta = \dfrac{2\sqrt{3}}{1 - e}$ where e is the coefficient of restitution between the spheres.

(ii) Show that the maximum percentage loss in kinetic energy due to the impact is $12\frac{1}{2}$%.

(iii) For what value of e will the kinetic energies of A and B after impact be in the ratio 7 : 1 ?

5. (a) Two cars A and B are moving along straight roads which are at right angles to each other, with uniform velocities 3 m/s and 4 m/s, respectively. When B is at the crossroads, A is 100 m away.
Calculate the time interval for which the distance between the cars is not greater than 82 m.

(b) A car of mass 750 kg attains a maximum speed of 30 m/s when travelling down an incline of 1 in 25 with the engine switched off. It can attain a maximum speed of 20 m/s up the same incline when the engine is working. The resistance to motion in each case is proportional to the square of the speed. Find

(i) the power at which the engine is working

(ii) the maximum speed of the car along a level road, if it works at the same power and its resistance is again proportional to the square of the speed.

6. Define simple harmonic motion.
A body of mass 0·25 kg hangs from a spiral spring. When pulled down 10 cm below its equilibrium position and released, it vibrates with simple harmonic motion of period 2 s.

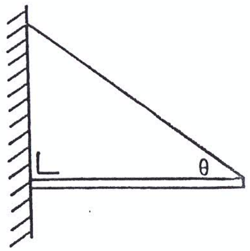
(i) Find its velocity as it passes through the equilibrium position.

(ii) What is the shortest time taken to travel from a point 2 cm below the equilibrium position to a point 2 cm above the equilibrium position ?

(iii) Find the elastic constant of the spring.

(iv) By how much will the spring shorten when the body is removed ?

7. One end of a uniform metre stick of mass m is placed against a vertical wall. The other end is held by a light inelastic string making an angle θ with the metre stick. The coefficient of friction, μ, between the end of the metre stick and the wall is 0·4.



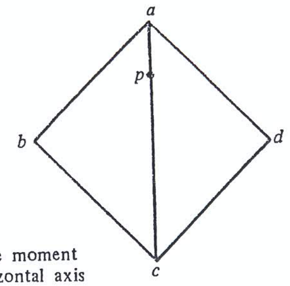
(i) Show in a diagram the forces acting on the metre stick.

(ii) Show that if the metre stick is to remain in equilibrium the maximum value of θ is given by $\tan\theta = \mu$.

(iii) If a mass m is suspended from the metre stick at a distance x from the wall, show that the stick is on the point of slipping when

$$\tan\theta = \frac{2(1 + 2x)}{5(3 - 2x)}.$$

8. A uniform square lamina $abcd$, of mass $3m$ and side $\sqrt{2}$, is free to rotate with its plane vertical about a smooth horizontal axis through a point p on the line ac. A mass m is attached at each of the points a and c.



(i) If $|ap| = 1 - x$, prove that the moment of inertia of the system about a horizontal axis through p is $m(3 + 5x^2)$.

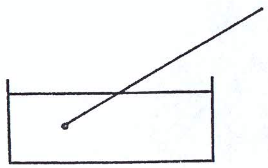
(ii) If the system oscillates about p, find in terms of x, the period for small oscillations.

(iii) Find the value of x which gives the minimum period when oscillations are small.

9. (a) A piece of gold-aluminium alloy of mass 10 kg, weighs 72 N in water. If the relative densities of gold and aluminium are 19·6 and 2·45 respectively, find

(i) the relative density of the alloy
(ii) the mass of each metal in the alloy
(iii) what fraction of the total volume of the alloy is gold.

(b) A uniform rod of relative density 0·25 is free to turn about its lower end, which is fixed at a depth 0·4 m in water. The rod is in equilibrium when partially immersed and making an angle of 60° with the vertical.
Find the length of the rod.



10. (a) Find the solution of the differential equation

$$3y^2(x - 1)\frac{dy}{dx} = 1 + y^3$$

if $y = 0$ when $x = 2$.

(b) A particle of mass m moves in a straight line. The only force acting on it being a resistance mkv^2, where v is its speed and k is a constant. It is initially projected from the point o with speed u. When the particle reaches a point p on the line its speed is $u/3$.

(i) Show that the average speed between o and p is $\frac{1}{2}u \log 3$.

(ii) Find the speed of the particle when it is at the midpoint of $[op]$.

LEAVING CERTIFICATE EXAMINATION, 1984

APPLIED MATHEMATICS – HIGHER LEVEL

WEDNESDAY, 27 JUNE – AFTERNOON, 2.00 – 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ m/s^2.

1. The driver of a car travelling at 20 m/s sees a second car 120 m in front, travelling in the same direction at a uniform speed of 8 m/s.

(a) What is the least uniform retardation that must be applied to the faster car so as to avoid a collision ?

(b) If the actual retardation is 1 m/s^2, calculate

(i) the time interval, in seconds, for the faster car to reach a point 66 m behind the slower.
(ii) the shortest distance between the cars.

2. A ship B is travelling in a direction 41° East of North at 15 m/s. A second ship C is travelling 41° South of East at 20 m/s.

Calculate: (i) the velocity of B relative to C;
(ii) the shortest distance between the ships if C is 3 km east of B at a particular moment ;
(iii) the time interval during which the ships remain in visual contact, if visibility is limited to 3 km.

3. A plane is inclined at an angle $\tan^{-1}(\frac{1}{2})$ to the horizontal. A particle is projected up the plane with velocity u at an angle θ to the plane. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane parallel to the horizontal.

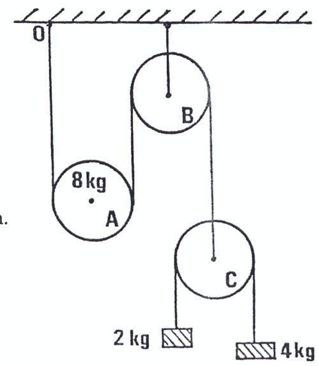
Express t, the time of flight, in terms of u and θ.

Hence, or otherwise, establish that

$$\tan\theta = \frac{1}{3}.$$

Calculate the range along the plane.

4. The diagram shows a light inextensible string having one end fixed at O, passing under a movable pulley A of mass 8 kg and then over a fixed light pulley B.
The other end of the string is attached to a light pulley C, of negligible mass.
Over pulley C, a second light inextensible string is passed having particles of mass 2 and 4 kg respectively, attached. All pulleys are smooth.



(i) Show in a diagram the forces acting on each pulley when the system is released from rest.

(ii) Find the acceleration of
pulley A
pulley C
each particle.

5. (a) A smooth sphere of mass 3 kg and velocity u_1 collides directly with another smooth sphere of mass 4 kg and velocity u_2 both moving in the same direction. Show that

$$7v_1 = u_1(3 - 4e) + 4u_2(1 + e)$$

where v_1 is the velocity of the 3 kg sphere after the collision.

Hence, show that the impulse which each sphere receives is

$$\tfrac{12}{7}(1 + e)(u_2 - u_1).$$

(b) A smooth sphere of mass 4 kg collides obliquely with another smooth sphere of mass m which is at rest. After impact the two spheres move at right angles to each other. If the coefficient of restitution was $\frac{4}{7}$, calculate the value of m.

6. (a) A particle moving on the inside smooth surface of a fixed hollow sphere of internal radius $\sqrt{2}$ m describes a horizontal circle of radius 1 m. Calculate the angular velocity of the particle.

(b) Two particles of equal mass attached by a taut inextensible string of length $2y$ rests on a horizontal circular table. The particles are respectively y and $3y$ from the centre of the table so that centre and particles are collinear.

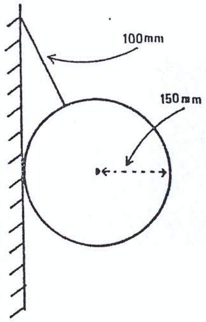
The table rotates about its centre with angular velocity ω and the coefficient of friction is $\frac{y}{2}$. If both particles are on the point of slipping,

(i) show on a diagram, all the forces of the string/particle system;

(ii) calculate ω.

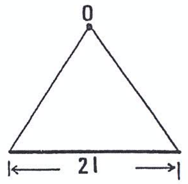
7. (a) A sphere of mass 3 kg and radius 150 mm is suspended by a string 100 mm long, the string joining a point on the surface with a point on a smooth vertical wall.

Find the tension in the string in terms of g.



(b) A heavy uniform rod of mass m and length $2l$ is suspended from a point, o, by two equal inelastic strings. Each string is fixed to o and to an end point of the rod so that the rod hangs horizontally.

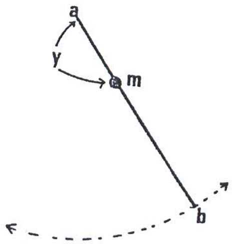
If, then, a mass $\frac{m}{2}$ is suspended half-way between the centre and one end of the rod so that the rod is no longer horizontal, calculate the ratio $T_1 : T_2$, where T_1 is the tension in one of the strings and T_2, the tension in the other.



8. A uniform rod $[ab]$ of length $2p$ and of mass $3m$ has a mass m attached to it at a distance y from a. Prove that the moment of inertia of this system about a smooth horizontal axis through a is

$$4mp^2 + my^2.$$

The system oscillates in a vertical plane about a. If the length of the equivalent simple pendulum is $\frac{40}{33}p$, show that y is either $\frac{2}{3}p$ or $\frac{6}{11}p$.



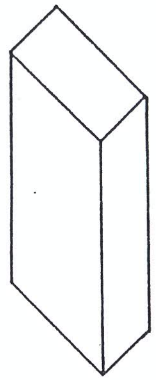
9. (a) A body of mass 1·5 kg weighs 2·1 N in water and 3·36 N in a mixture of another liquid A and water. If there was no reduction in volume when the mixture was made, calculate

(i) the relative density of the body

(ii) the relative density of the mixture

(iii) the volume of liquid A, of relative density 0·82, which must be added to 100 ml of water to form the mixture.

(b) A uniform rectangular block of wood 200 mm X 100 mm X 8 mm and of relative density d floats in water with its longest edge vertical. If the block is depressed vertically a further small distance x and released, verify that it will perform simple harmonic motion. Calculate the periodic time of the motion.



10. (a) Find the general solution to

$$\frac{dv}{dt} = g - kv$$

where g and k are constants.

Show that $\lim_{t \to \infty} v = \frac{g}{k}$.

(b) A car, free-wheeling on a straight road, experiences a retardation which is proportional to the square of its speed. Its speed is reduced from 20 m/s to 10 m/s in a distance of 100 m. Calculate the time taken to travel the 100 m.

LEAVING CERTIFICATE EXAMINATION. 1983

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE—AFTERNOON, 2.00–4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.

1. A train of length 120 m has an acceleration of 1 m/s^2. It meets another train of length 80 m travelling on a parallel track in the opposite direction with an acceleration of 1·5 m/s^2. Their speeds at this moment are respectively 20 m/s and 25 m/s. Show, by diagrams, the positions of the trains just before meeting and immediately after passing.
Find the time taken for the trains to pass each other.
If one of the trains, by applying brakes, were to cause an increase of $12\frac{1}{2}$% in this time of passing, calculate to the nearest m/s^2 the decrease in its acceleration.

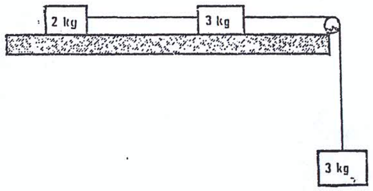
2. An aircraft flew due east from p to q at u_1 km/h. Wind speed from the south west was v km/h. On the return journey from q to p, due west, the aircraft's speed was u_2 km/h, the wind velocity being unchanged. If the speed of the aircraft in still air was x km/h, $x > v$, show, by resolving along and perpendicular to pq, or otherwise, that

(i) $u_1 - u_2 = v\sqrt{2}$

(ii) $u_1 u_2 = x^2 - v^2$

If $|pq| = d$, find in terms of v, x and d, the time for the two journeys.

3. The diagram shows particles of mass 2 kg and 3 kg, respectively, lying on a horizontal table in a straight line perpendicular to the edge of the table. They are connected by a taut, light, inextensible string. A second such string passing over a fixed, light pulley at the edge of the table connects the 3 kg particle to another of mass 3 kg hanging freely under gravity. The contact between the particles and the table is rough with coefficient of friction $\frac{1}{4}$.
Show in separate diagrams the forces acting on the particles when the system is released from rest. Calculate



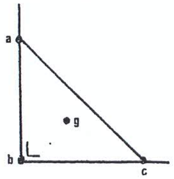
(i) the common acceleration

(ii) the tension in each string in terms of g.

4. A bullet is fired from a gun fixed at a point o with speed v m/s at an angle of θ to the horizontal.
At the instant of firing, a moving target is 10 m vertically above o and travelling with constant speed $42\sqrt{2}$ m/s at an angle of 45° to the horizontal.
The bullet and target move in the same plane.
If v=70 m/s and tan θ=4/3, find at what time after firing does the bullet strike the target and calculate the horizontal distance of the bullet from o.
Show that the minimum value of θ to ensure that the bullet strikes the target is given by tan θ = 4/3.

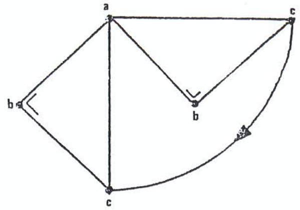
5. State the laws governing oblique collisions between two smooth elastic spheres.
Two such spheres A and B of mass 5 and 10 kg respectively, collide obliquely. The coefficient of restitution is $\frac{1}{7}$.
Immediately before collision the velocity of A is $5\vec{i} + 4\vec{j}$ and that of B is $-2\vec{i} - 3\vec{j}$, where speeds are in m/s and $\vec{i}$ and $\vec{j}$ are unit vectors along and penpendicular to the line of centres.
Find the velocity of (i) A (ii) B immediately after impact.
Show that the loss of kinetic energy is 80 J.
Calculate the tan of the angle through which B is deflected after the collision.

6. (i) A uniform triangular lamina abc is of mass m with $|ab| = |bc| = 6$ and $|\angle abc| = 90°$. Show that its moment of inertia about bc is $6m$.



(ii) Prove that the moment of inertia of abc about an axis through a perpendicular to the plane of abc is $24m$. [Coordinates of g, the centre of gravity of abc is (2,2) when the origin is at b]

(iii) The axis through a is fixed horizontally so that the lamina can rotate freely under gravity in a vertical plane. It is released from rest with ac horizontal and above b. Find, in terms of g, the speed of c when ac is vertical.



7. A hollow right circular cone of semivertical angle α, where tan $\alpha = \frac{1}{3}$, is fixed with its axis vertical and vertex downwards. The inner surface of the cone is rough with coefficient of friction $\frac{1}{2}$ and the cone rotates about its axis with uniform angular velocity 7 rad/s.
A particle of mass m is placed on the inside surface and rotates with the cone at a vertical height h above the vertex. Calculate the normal reaction of the particle with the inside surface and the height h above the vertex if

(i) the particle is about to slide down

(ii) the particle is about to slide up.

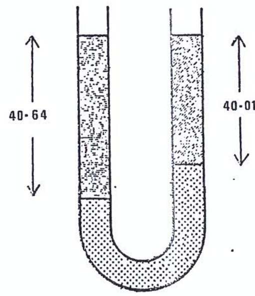
8. Define simple harmonic motion in a straight line and show that

$$x = a \sin \omega t$$

can describe such motion, when x is the distance from a fixed point and a, ω and t have the usual meanings.
A particle p, of mass 5 kg, is connected by a light elastic string, of natural length 2 m and elastic constant 140 N/m to a fixed point q on a rough horizontal surface where the coefficient of friction is 1.
p is released from rest at a point a where $|qa| = 3$ m.
By considering the forces acting on p when its distance is $(2{\cdot}35 + x)$m from q, prove that p moves in simple harmonic motion as long as the string remains taut. State the position of the centre, o, of the simple harmonic motion i.e. $|qo|$ and write down the amplitude.

If the periodic time is assumed to be $\frac{\pi}{\sqrt{7}}$ calculate the time taken by the particle to travel from a to a point 2 m from q.

9. (a) Mercury occupies the curved portion of a fixed upright U-tube of uniform cross-section. Water is poured into one arm and alcohol into the other, until both free surfaces are at the same level. The lengths of water and alcohol columns are 40·64 cm and 40·01 cm respectively. Calculate the density of alcohol.



More alcohol is then poured in until the two mercury surfaces are at the same level. Find the new length of the alcohol column to the nearest mm. [Density of water = 1000 kg/m^3. Relative density of mercury = 13·6]

(b) A right circular cone of base radius r and vertical height $3r$ is held submerged with its vertex downwards in a liquid of density ρ. Its plane base is horizontal and is at a distance r below the surface.
Calculate the forces exerted by the liquid on

(i) the base
(ii) the curved surface of the cone.

10. (a) Find the solution of the differential equation

$$\sin x \frac{dy}{dx} = y \cos x$$

when $y = 2$ at $x = \frac{\pi}{6}$.

(b) A particle of mass 8 kg moves along a line (the x-axis) on a smooth horizontal plane under the action of a force in newtons of

$$(40 - 3\sqrt{x})\vec{i}$$

where $\vec{i}$ is the unit vector along the axis and x is the distance of the particle from a fixed point o of the axis.
If the particle starts from rest at o, find its speed when $x = 100$ and calculate when it next comes to instantaneous rest.